Written exam on Lab-on-a-Chip course, Spring Semester 2009
June 8th, 2009

Examination time: 4 hours (9am – 1pm)
Allowed means: MYO "Fundamentals of Fluid Mechanics", lecture slides and a calculator (no computer or PDAs, please)
Complete solution should include all equations calculated down to a numerical answer. Numerical answer alone is not counted as a solution.
Some useful constants for your problems are listed in the end of the exam paper

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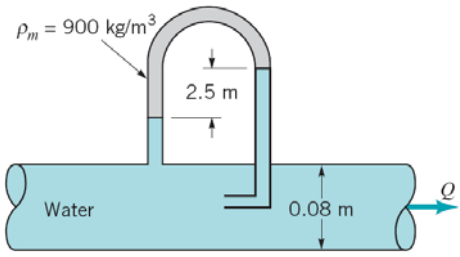
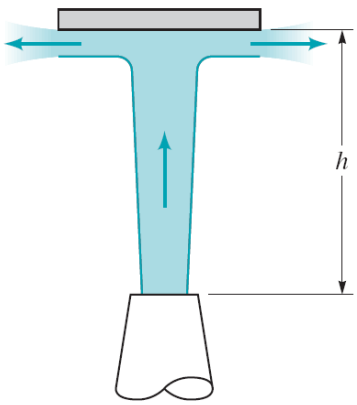
Problem 1. Determine flow rate through the pipe shown in the figure.



Problem 2. A vertical jet of water leaves a nozzle at a speed of 10 m/s and a diameter of 20mm. It suspends a plate of 1.5 kg. Find the distance h.



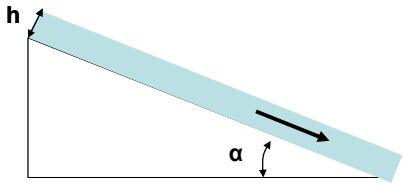
Problem 3. The velocity components of a 2D velocity field are given by the equations:

$u = y^2 - x(1+x)$

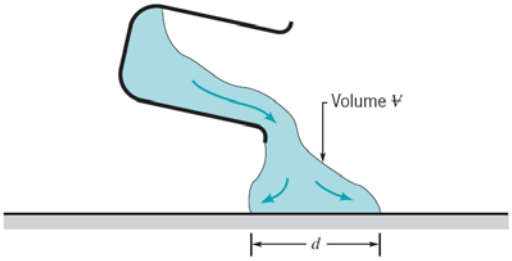
$v = y(2x+1)$

(a) Find the stream function
(b) Find the acceleration
(c) Show that the flow is irrotational and satisfies the conservation of mass

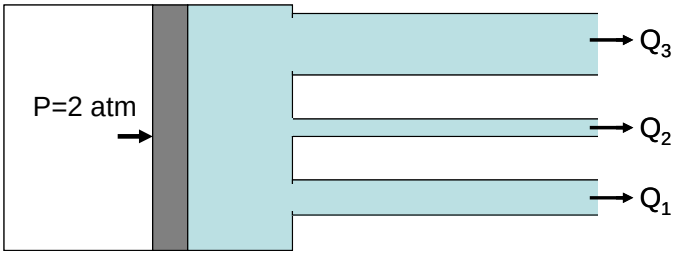
Problem 4. Using Navier-Stokes equation, determine relation between the volumetric flow rate and the height **h** of the layer of viscous liquid of constant thickness flowing steadily down an infinite inclined plane (inclination angle **α**). Assume laminar flow and negligible air resistance.



Problem 5. A viscous liquid is poured onto a horizontal plate as shown in the figure. Assume the time ***t*** required for the fluid to flow a certain distance ***d*** along the plate is a function of a volume poured ***V***, acceleration of gravity ***g***, fluid density ρ and viscosity ***μ***. Determine the set of Pi-terms to describe the process.



Problem 6. Commonly in "industrial" microfluidics, the liquid is propelled by air pressure that can be switched On and Off with a simple valve. In a circuit shown in the figure, the liquid flows through three rectangular channels 50μm, 100 μm and 200 μm wide. The height of the channels is equal to 50 μm and the length is 1 cm. Calculate the volumetric flow rate through each channel at a pressure of 2 atm. What will be the Re numbers for each channel at this condition?



Problem 7. Electroosmotic effect is used to propel 10mM NaCl solution through a channel that is 200um wide, 50um high and 20mm long. What voltage should be applied between the inlet and the outlet to achieve flow through the channel equal to 1ul/min. Use approximation of flow between two infinite parallel plates, zeta potential on the wall is -100mV

List of constants:

Density of water 1000 kg/m^3;
Viscosity of water $1 \cdot 10^{-3}$ Pa*s
Permittivity of vacuum: $\varepsilon_0 = 8.854 \cdot 10^{-12}$ $C/(N \cdot m^2)$.
Relative permittivity of water $\varepsilon = 78$